## Structure of arithmetic pro-p fundamental groups and applications

## William McCallum

April 15, 2004

Let p be an odd prime, K a number field containing the group  $\mu_p$  of pth roots of unity, and G the Galois group of the maximal pro-p extension of K unramified outside a finite set S of primes containing the primes above p and the archimedean primes. The author and Romyar Sharifi studied the cup product pairing on  $H^1(G, \mu_p)$  that regulates the quadratic term in the relations between generators of G, and, in the case  $K = \mathbb{Q}(\mu_p)$ , calculated the pairing up to a scalar multiple for primes p < 10,000. More recently, Sharifi has shown the scalar to be nonzero for many primes, thus showing the pairing to be nontrivial in those cases. In this talk we describe applications of this nontriviality, including joint work with Ben Levitt relating the pairing to the Cassels pairing on the Shafarevich-Tate group of the Jacobian of a Fermat curve. We describe the consequences of non-triviality of the pairing for higher descents on the Jacobian.